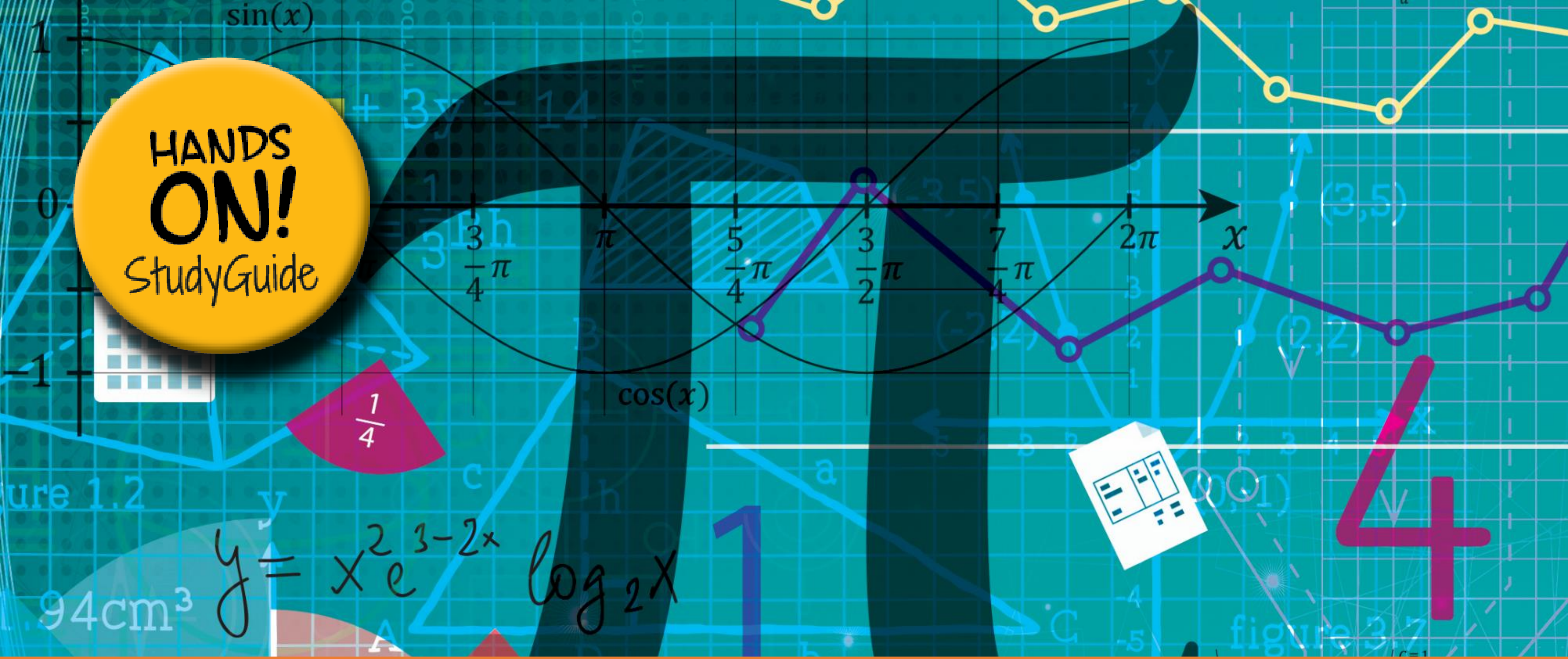


Mathematics
N4

Module 1: Equations, manipulations and word problems

Factorise $a^3 \pm b^3$

To solve, let a and b be real numbers, variables, or algebraic expressions.

1. Write as sum or difference of two cubes.
2. Write in the factored form.
3. Simplify the factored form

THE BASIC OPERATIONS &THE LAWS OF INDICES AND LOGARITHMS

Solving exponential formulas:

1. Isolate the exponential expression.
2. Apply the logarithm to each side of the formula, and then remove the variable in the exponent by using Logarithmic Laws.
3. Solve for the variable.

Solving logarithmic formulas:

1. Isolate the logarithmic term.
2. Transform the logarithmic formula to an exponential formula.
3. Solve for the variable.

SOLVE THREE SIMULTANEOUS LINEAR EQUATIONS FOR THREE UNKNOWN VARIABLES

Strategy for solving a system of linear equations in three variables using the elimination method

Step 1 Write all the equations in the standard form $Ax + By + Cz = D$, as this will ease the elimination of variables.

Step 2 Select a variable to eliminate. Choose any two equations and eliminate the selected variable. This creates an equation with two unknowns.

Step 3 Choose another two equations and eliminate the same variable selected in step 2. This also creates an equation with the same two unknowns as in step 2.

Step 4 Solve by means of elimination the two equations from step 2 and step 3. This will result in the solutions of two variables.

Step 5 Select the set of three variable equations in step 2 and step 3, and eliminate one of the variables solved in step 4 for each of the sets to form two equations with two unknowns.

Step 6 Solve for the third variable using the two equations with two unknowns from step 5.

Step 7 Check the solution with all three original equations.

Strategy for solving a system of linear equations in three variables using the substitution method

Step 1 Select any one of the equations and isolate one variable.

Step 2 Substitute the solved variable in step 1 into any one of the two remaining equations, and solve for the next variable.

Step 3 Substitute the solved variable in step 2 into the solved variable in step 1.

Step 4 Substitute the variables solved in step 2 and step 3 into the last remaining equation and solve the value of the variable.

Step 5 Substitute the value of the variable in step 4 into step 2 and solve for the value of the variable.

Step 6 Substitute the value of the variable in step 4 into step 3 and solve for the value of the last variable.

Step 7 Check your solution by substituting the proposed solution into all three equations.

SOLVE REAL-LIFE WORD PROBLEMS IN EQUATIONS

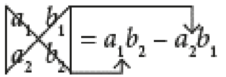
When an answer is needed to a mathematical problem that is described in words as well as numbers, it is called a word problem. You find the solution by setting up an equation that represents the problem, and then solve it.

Module 2: Determinants

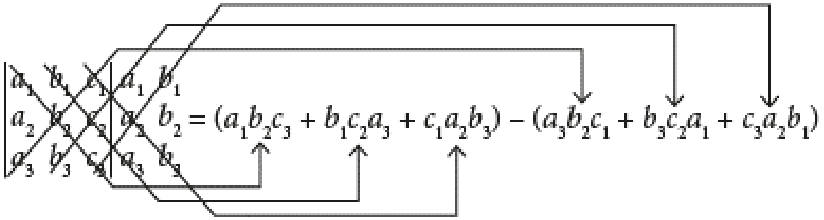
WRITE SIMULTANEOUS EQUATIONS IN DETERMINANT NOTATION AND EVALUATE BY APPLYING SARRUS' RULE

First write the equations in the standard form. The coefficients of the variables will be captured to form the determinants. Then, a second and third order determinant will be evaluated using Sarrus' rule:

Sarrus' rule for a second order determinant



Sarrus' rule for a third order determinant



IDENTIFY AND CALCULATE THE MINOR OF A THIRD ORDER DETERMINATION

The Minor Expansion of a third order determinant requires the expansion of a row or column. In terms of the minor expansion, the third order determinant is defined as the sum of the product of the sign from the sign array for the specific entry, the entry and the minor of the entry.

DEFINE AND DETERMINE THE COFACTOR OF THE MINOR

The cofactor of a minor is given as:

$$C_{ij} = (-1)^{i+j}.M_{ij}$$

where i and j are the respective row and column of the element, and M_{ij} the minor of the element.

CALCULATE DETERMINANTS OF THE SECOND AND THIRD ORDER AND APPLY CRAMER’S RULE

Cramer’s rule is an alternative method to solving a system of linear equations in three variables compared to the methods of substitution and elimination.

Module 3: Complex numbers

WRITE COMPLEX NUMBERS IN THE RECTANGULAR (STANDARD) FORM

A complex number can be represented in the rectangular form, that is

$z = a + bi$

where a and b are real numbers, and $\mathrm{i} = \sqrt{-1}$ is the imaginary unit with the property $i^2 = -1$.

WRITE COMPLEX NUMBERS IN THE POLAR (TRIGONOMETRIC) FORM.

A complex number can be represented in the polar form, that is

$z = r(\cos\theta + i\sin\theta)$

where r is the modulus, the distance from the origin to a point $(a; b)$ and θ is the argument, the positive angle measured from the positive real axis anticlockwise.

SOLVE A QUADRATIC EQUATION WITH COMPLEX ROOTS

Steps for solving a quadratic equation with complex roots:

1. Write the quadratic equation in the standard form, $ax^2 + bx + c = 0$.
2. Record the numeric values of a (coefficient of x^2), b (coefficient of x) and c (constant).
3. Substitute the numeric values of a, b and c into the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and write the solutions in standard (rectangular) form.

SOLVE COMPLEX EQUATIONS

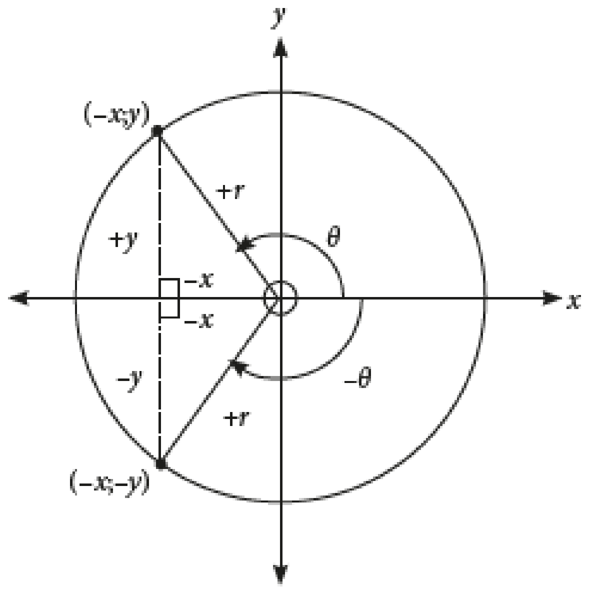
To solve:

1. Write both sides of the complex equation in the standard (rectangular) form, $a + bi = c + di$.
2. Equate the real parts, $a = c$ and imaginary parts, $b = d$.
3. Compute the unknown variables.
4. Verify the solution.

Module 4: Trigonometry

APPLY THE CONCEPT OF NEGATIVE AND POSITIVE ANGLES TO ALL THE CALCULATIONS RELEVANT TO THE SYLLABUS

Measuring angles from the positive x –axis in a counter clockwise rotation yields a positive angle and the clockwise rotation yields a negative angle.



APPLY THE IDENTITIES FOR $sin(a \pm b)$, $cos(a \pm b)$, AND $tan(a \pm b)$.

The sum and difference identities are also referred to as the compound angle or composite angle identities.

Strategy for using the sum and difference identities:

1. Rewrite the angle as a compound angle, that is, $a \pm b$.
2. Expand the appropriate compound angle identity, that is, sine, cosine and tangent.

DERIVE THE IDENTITIES FOR $sin2x$, $\cos 2x$, $tan2x$, $sin^2 x = \frac{1}{2}(1 - cos2x)$, $\cos^2 x = \frac{1}{2}(1 + cos2x)$, $sin\frac{1}{2}x$, $\cos\frac{1}{2}x$ AND $tan\frac{1}{2}x$ AND APPLY

The double angle identities are derived from the compound angle identities.

This gives:

Double angle identities
$\sin 2x = 2\sin x.\cos x$
$\cos 2x = \cos^2 x - \sin^2 x$ $\cos 2x = 1 - 2\sin^2 x$ $\cos 2x = 2\cos^2 x - 1$
$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

DERIVE THE CO-RATIOS $sin(90° \pm x)$, $\cos(90° \pm x)$, AND $\tan(90° \pm x)$ AND APPLY

The compound angle identities can also be used to derive co-function identities.

This gives:

Co-function identities		
$\sin(90° - x) = \cos x$	$\cos(90° - x) = \sin x$	$\tan(90° - x) = \cot x$
$\sin(90° + x) = \cos x$	$\cos(90° + x) = -\sin x$	$\tan(90° + x) = -\cot x$

APPLY THE IDENTITIES TO SOLVE TRIGONOMETRIC EQUATIONS:

1. Rewrite the equation in terms of one trigonometric function if possible by using trigonometric identities.
2. Determine whether the equation is linear or quadratic.
3. Apply factorising techniques.
4. Solve the trigonometric equation and verify the solutions with the desired interval.

APPLY THE IDENTITIES TO SIMPLIFY TRIGONOMETRIC EXPRESSIONS

1. Simplify the individual functions by using the trigonometric knowledge outlined.
2. Rewrite the simplified trigonometric functions in terms of sine and cosine.
3. Simplify the sine and cosine trigonometric functions.

APPLY THE IDENTITIES TO PROVE TRIGONEMTRIC IDENTITIES

1. Identify the more complicated side.
2. Simplify the trigonometric functions.
3. Rewrite the simplified trigonometric functions in Step 2 in terms of sine and cosine.
4. Simplify the sine and cosine trigonometric functions in Step 3.
5. Verify whether the selected side in Step 1 is equal to the other side.

DRAW TRIGONOMETRIC SKETCH GRAPHS OF $y = asin(bx +) = d$; $y = acos(bx +) = d$ AND $y = atan(bx +) = d$ for $-2\pi \leq x \leq 2\pi$

1. Sketch the basic trigonometric graph.
2. Identify the amplitude a and modify the basic graph.
3. Identify the frequency b and modify the graph in Step 2.
4. Identify the phase angle $\frac{c}{d}$ and modify the graph in Step 3.
5. Identify the vertical shift d and modify the graph in Step 4.
6. Modify the graph to fit the given domain.

DRAW RECIPROCAL TRIGONOMETRIC SKETCH GRAPHS OF

$y = cosecx, y = secx$ AND $y = cotx$ FOR $-2\pi \leq x \leq 2\pi$

1. Write the reciprocal trigonometric equation in terms of sin$x$, cos$x$ or tan$x$.
2. Compute points by means of the table method.
3. Create a Cartesian coordinate system.
4. Draw the reciprocal trigonometric graph and clearly show the asymptotes, and turning points and x-intercepts where applicable.

Module 5: Sketch graphs

IDENTIFY CHARACTERISTICS IN RESPECT OF FUNCTIONS AND NON-FUNCTIONS

- Domain and range;
- Independent and dependent variable;
- Functions and non-functions;
- Points of symmetry with reference to an axis or the lines $y = \pm x$;
- Continuous and discontinuous functions or non-functions; and
- Inverse functions and non-functions.

DRAW NEAT SKETCH GRAPHS OF FUNCTIONS AND NON-FUNCTIONS

- Straight line graph $y = mx + c$;
- Circle and semi-circle graph $x^2 + y^2 = r^2$;
- Rectangular hyperbola $xy = c$;
- Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$;
- Centred hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$;
- Exponential and logarithmic graph $y = ka^{nx}$ and $y = klog_a(nx)$; and
- Parabola $y = ax^2 + bx + c$.

Module 6: Limits and differentiation

APPLY THE THEOREMS ON LIMITS TO CALCULATE LIMITS USING ALGEBRAIC EXPRESSIONS AND QUOTIENTS

Several different techniques are used to evaluate limits of determinate forms.

The strategy for using limits is:

1. Evaluate limits by applying direct substitution.
2. If Step 1 yields indeterminate forms $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then apply one of the remaining techniques.

GIVE THE BINOMIAL THEOREM IN GENERAL TERMS

The general term of the binomial expression is:

$$T_{r+1} = \binom{n}{r} x^{n-1} h^r$$

APPLY THE BINOMIAL THEOREM WITH RATIONAL INDICES TO EXPAND A SIMPLE BINOMIAL TO FOUR TERMS

Strategy for using the generalised version of the binomial theorem:

1. Write the expression in the form $(x+h)^n$.
2. Expand the expression using the generalised version of the binomial theorem.
3. Simplify the expansion.

DEFINE DIFFERENTIATION AS RATE OF CHANGE AND DERIVE THE EXPRESSION $\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$ OR $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ FROM FIRST PRINCIPLES WITH THE AID OF A SKETCH

Strategy for determining the derivative from first principles:

1. Replace x in the given function $f(x)$ with $x+h$, that is, $f(x+h)$.
2. Simplify the numerator, that is, $f(x+h) - f(x)$.
3. Remove the common factor h and simplify $\frac{f(x+h)-f(x)}{h}$ by cancelling the h.
4. Substitute $h = 0$ and simplify

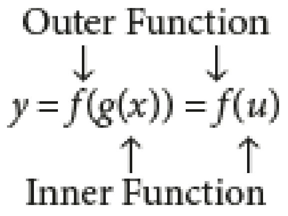
DETERMINE $\frac{dy}{dx}$ OF THE STANDARD FORMS: $y = k$; $y = kx^n$; $y = ka^x$; $y = ke^x$; $y = k\ln kx$; $y = k\log_a x$;$y = ksinx$; $y = kcosx$; $y = ktanx$; $y = kcotx$; $y = ksecx$, AND $y = kcosecx$.

Strategy for determining derivatives of standard functions:

1. Rewrite the original function in the format of the standard differentiation formulas.
2. Differentiate the rewritten function.

APPLY THE CHAIN RULE TO A FUNCTION OF A FUNCTION TO DETERMINE THE FIRST DERIVATIVES.

1. Substitute $u = nx$ for the composite functions, and rewrite the composite functions in terms of u.
2. Compute $\frac{du}{dx}$ and $\frac{dy}{du}$.
3. Multiply $\frac{dy}{du}$ and $\frac{du}{dx}$ to find $\frac{dy}{dx}$.
4. Rewrite the resulting derivative in terms of x and simplify.



APPLY THE DERIVATIVES TO FIND THE FIRST DERIVATIVES OF POLYNOMIALS

Differentiation of polynomials	
Sum Rule for Differentiation	$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$
Difference Rule for Differentiation	$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$

Strategy for determining derivatives of polynomials:

1. Rewrite the derivative of a polynomial as separate derivatives by using the sum and/or difference rule for differentiation.
2. Differentiate each individual function.

APPLY THE PRODUCT AND QUOTIENT RULES FOR DIFFERENTIATION TO DIFFERENTIATE SIMPLE PRODUCTS AND QUOTIENTS

Strategy for determining the Product rule and Quotient rule:

1. Select $f(x)$ and $g(x)$, and compute $f'(x)$ and $g'(x)$.
2. Substitute $f(x)$, $g(x)$, $f'(x)$ and $g'(x)$ into the Product or Quotient rule.
3. Simplify the Product rule or Quotient rule.

Product rule
$y = f(x).g(x)$ $\therefore \frac{dy}{dx} = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$

Quotient rule
$y = \frac{f(x)}{g(x)}$ $\therefore \frac{dy}{dx} = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{(g(x))^2}$

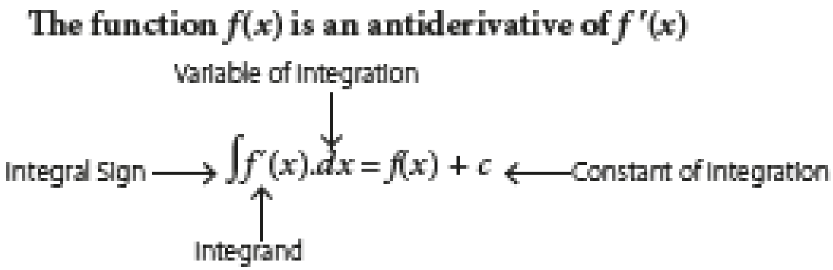
Module 7: Integration

UNDERSTAND THE CONCEPT OF INTEGRATION AS A SUMMATION FUNCTION AND AS THE INVERSE OF DIFFERENTIATION

Differentiation and integration (antiderivative) are inverse processes. The integration is denoted by an integral sign $\int$.

$\int f'(x).dx = f(x) + c$	Integration is the inverse of differentiation.
$\frac{d}{dx}\left[\int f'(x).dx\right] = f'(x)$	Differentiation is the inverse of integration.

APPLY STANDARD FORMS OF INTEGRALS AS THE INVERSE OF DIFFERENTIATION



Strategy for determining integrals of standard functions:

1. Rewrite the original integral in the standard integration formulas format.
2. Integrate the rewritten integral and include a constant of integration.
3. Verify the solution by differentiating.

INTEGRATE COMPOSITE FUNCTIONS

Strategy for determining integrals of composite functions:

1. Substitute $u = nx$ for the composite functions $k.a^{nx}$, $k.e^{nx}$ and $u = bx$ for the composite functions $k.\sin(bx)$ and $k.\cos(bx)$.
2. Compute $du = n.dx$ or $du = b.dx$. and rewrite the integral in terms of u.
3. Integrate in terms of u and include the constant of integration.
4. Rewrite the resulting integral in terms of x and verify the solution by differentiating.

INTEGRATE POLYNOMIALS

Integration of polynomials	
Sum rule for integration	$\int [f(x) + g(x)].dx = \int f(x).dx + \int g(x).dx$
Difference rule for integration	$\int [f(x) - g(x)].dx = \int f(x).dx - \int g(x).dx$

Strategy for determining integrals of polynomials

1. Rewrite the integral of a polynomial as separate integrals by using the sum and/or difference rule for integration.
2. Integrate each individual integral.
3. Verify the solution by differentiating.

CALCULATE DEFINITE INTEGRALS OF FUNCTIONS

The definite integral is an extension of the indefinite integral, that is, the evaluation of the antiderivative that yields a number.

Strategy for determining definite integrals:

1. Rewrite the original integral in the standard integration formulas format.
2. Integrate the rewritten integral.
3. Evaluate the antiderivative, that is, the upper limit evaluation minus the lower limit evaluation.

DETERMINE THE MAGNITUDE OF AN AREA INCLUDED BY A CURVE AND THE x-AXIS, OR BY A CURVE, THE x-AXIS AND THE ORDINATES $x = a$ AND $x = b$, WHERE a AND b ARE INTEGERS

Strategy for determining area by means of integration:

1. Identify whether the shaded required area(s) are above the x-axis.
2. Indicate the representative strip and limits of integration.
3. Write down the area of representative strip, that is, $\Delta A = y.\Delta x$
4. Compute the total area by using the definite integral.